NT@UW-98-19
CALT-68-2175
DUKE-TH-98-167
DOE/ER/40561-17-INT98

An Effective Field Theory Calculation of the Parity Violating Asymmetry in $\vec{n} + p \to d + \gamma$

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Abstract

Weak interactions are expected to induce a parity violating pion-nucleon coupling, $h_{\pi NN}^{(1)}$. This coupling should be measurable in a proposed experiment to study the parity violating asymmetry A_γ in the process $\vec{n} + p \to d + \gamma$. We compute the leading dependence of A_γ on the coupling $h_{\pi NN}^{(1)}$ using recently developed effective field theory techniques and find an asymmetry of $A_\gamma = +0.17\ h_{\pi NN}^{(1)}$ at leading order. This asymmetry has the opposite sign to that given by Desplanques, Donoghue and Holstein.

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Recently an improved measurement of the parity violating asymmetry, A_γ, in the angular distribution of 2.2 MeV gamma rays from the radiative capture of polarized cold neutrons $\vec{n} + p \to d + \gamma$, was proposed [1]. With $\theta_{\mathbf{s}\gamma}$ the angle between the neutron spin and the photon momentum, the asymmetry A_γ is defined by

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_{\mathbf{s}\gamma}} \ = \ 1 \ + \ A_\gamma \ \cos\theta_{\mathbf{s}\gamma} \quad . \tag{1}$$

The current experimental limit on this asymmetry parameter is $A_\gamma = -(1.5 \pm 4.8) \times 10^{-8}$ [2], while the proposed experiment expects to measure A_γ with a precision of $\pm 5 \times 10^{-9}$. Interest in A_γ is motivated by a recent measurement [3] of the cesium anapole moment that appears to give weak coupling parameters that are inconsistent with other low energy parity violating measurements [4,5]. In order to obtain the theoretically cleanest determination of the weak parameters in the nucleon-meson Lagrange density it is clear that measurements in few nucleon systems are desirable, thereby eliminating density effects that could arise in nuclei and are difficult to calculate. Reviews of the the subject can be found in [6–10].

A_γ is sensitive to the weak parity violating pion-nucleon coupling, and its measurement may provide a clean determination of this important parameter. In this letter we present a calculation of A_γ using recently developed effective field theory techniques [11,12]. This method allows us to calculate processes in the two nucleon sector in a systematic fashion, and if carried out to higher orders, is expected to be able to reach the same level of precision as conventional nuclear physics techniques, but without model dependence.

The effective field theory method was used to calculate the electromagnetic form factors of the deuteron in [12] and the basic tools for performing the calculations in this letter were developed there. The only new interaction needed is the parity violation pion-nucleon coupling which appears in the interaction Lagrange density,

$$\mathcal{L}_{pnc} = -\ \frac{h^{(1)}_{\pi NN}\epsilon^{3ij}}{\sqrt{2}} \ N^\dagger \ \tau^i \pi^j \ N \quad . \tag{2}$$

N is the doublet of spin $\frac{1}{2}$ nucleon fields, (i.e., $N_1 = p$ and $N_2 = n$), τ^i, $i = 1, 2, 3$ are the three Pauli matrices in isospin space and π^j, $j = 1, 2, 3$ are the three (real) pion fields. The $\Delta I = 1$ weak pion-nucleon coupling $h^{(1)}_{\pi NN}$ is simply related to the notation of Desplanques, Donoghue and Holstein (DDH) [7], $h^{(1)}_{\pi NN} = f_\pi$ [1], but is of opposite sign to that used in [13]. The strong pion-nucleon interaction Lagrange density we have used is

$$\mathcal{L}_{pc} = \frac{g_A}{f_\pi} N^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\nabla} \pi N \quad , \tag{3}$$

where $\pi = \tau^i \pi^i / \sqrt{2}$, $g_A = +1.25$ is the axial coupling constant and $f_\pi = 132$ MeV is the pion decay constant.

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[1] There are typographical errors in [6,10]. In eq. 3 of [6] and eq. 7 of [10] the replacement $\frac{f_\pi}{2} \to \frac{f_\pi}{\sqrt{2}}$ should be made. In this letter the symbol f_π is reserved for the pion decay constant. The signs of coupling constants used in [22] are consistent with those used in [7] only if the strong coupling is negative, in which case redefining all meson fields $M \to -M$ will give rise to the signs used in [7].

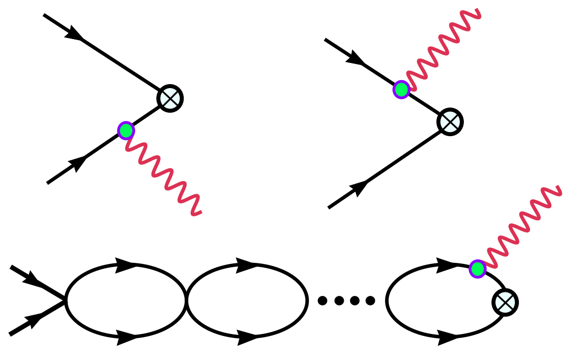

FIG. 1. *Graphs contributing to the parity conserving amplitude for $n + p \to d + \gamma$ at leading order in the effective field theory expansion. The solid lines denote nucleons and the wavy lines denote photons. The light solid circles correspond to the nucleon magnetic moment coupling to the electromagnetic field. The crossed circle represents an insertion of the deuteron interpolating field which is taken to have $^3S_1$ quantum numbers.*

Predictions based on effective field theory are made in a chiral and momentum expansion. For cold neutron capture $n + p \to d + \gamma$ with the neutron and proton essentially at rest the relevant momentum Q is determined by the binding energy of the deuteron, $B = 2.224$ MeV and is $\sqrt{M_N B} = 45.70$ MeV, where M_N is the nucleon mass. The power counting treats Q/Λ_{QCD} and m_π/Λ_{QCD} (where Λ_{QCD} is the scale characteristic of short range nucleon-nucleon interactions) as small and takes $Q \sim m_\pi$. This is similar in spirit to applications of chiral perturbation theory in the single nucleon sector and for pion self interactions. However, the power counting is unlike that used in conventional chiral perturbation theory because of the large scattering lengths that occur in the $^1S_0$ and $^3S_1$ NN scattering channels. These large scattering lengths render the leading order interactions nonperturbative and cause the two-body operators to develop large anomalous dimensions. The expansion is described in detail in Refs. [11,12,14]. At lowest order in the Q power counting $\mathcal{L}_{pnc}$, eq. (2), is the only parity violating interaction that occurs [13,14]. Other terms, such as the parity violating two-body operators are not relevant until higher order in the Q expansion [14].

At leading order in the Q expansion we find the matrix element for cold neutron capture $n + p \to d + \gamma$ can be written as

$$\begin{aligned}\mathcal{M} = & \; e\ X\ N^T \tau_2\ \boldsymbol{\sigma}_2\ \left[\boldsymbol{\sigma}\cdot\mathbf{q}\ \ \boldsymbol{\epsilon}(d)^* \cdot \boldsymbol{\epsilon}(\gamma)^*\ -\ \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(\gamma)^*\ \ \mathbf{q}\cdot\boldsymbol{\epsilon}(d)^*\right] N \\ & + ie\ Y\ \epsilon^{ijk}\ \boldsymbol{\epsilon}(d)^{i*}\ \mathbf{q}^j\ \boldsymbol{\epsilon}(\gamma)^{k*}\ (N^T \tau^2 \tau^3 \boldsymbol{\sigma}^2 N) \\ & + ie\ W\ \epsilon^{ijk}\ \boldsymbol{\epsilon}(d)^{i*}\ \boldsymbol{\epsilon}(\gamma)^{k*}\ (N^T \tau^2 \boldsymbol{\sigma}^2 \boldsymbol{\sigma}^j N) \quad .\end{aligned} \tag{4}$$

In eq. (4) $e = |e|$ is the magnitude of the electron charge, N is the doublet of nucleon spinors, $\boldsymbol{\epsilon}(\gamma)$ is the polarization vector for the photon, $\boldsymbol{\epsilon}(d)$ is the polarization vector for the deuteron and $\mathbf{q}$ is the outgoing photon momentum. The terms X and Y are parity conserving while the term W is parity violating. Note that for the parity conserving term Y the neutron and proton are in a $^1S_0$ state while for the parity conserving term X and the parity violating term W they are in a $^3S_1$ state. Interference between the parity conserving and parity violating amplitudes is possible if the neutron is polarized. At leading order, X and Y are calculated from the sum of Feynman diagrams in Fig. (1) and from wavefunction renormalization associated with the deuteron interpolating field [12], giving

$$X = 0 \quad , \quad Y = -\sqrt{\frac{\pi}{\gamma^3}}\, \frac{2\kappa_1}{M_N}\, (1-\gamma a_0) \quad , \tag{5}$$

where $\kappa_1 = \frac{1}{2}(\kappa_p - \kappa_n) = 2.35294$ is the isovector nucleon magnetic moment in units of nuclear magnetons, $a_0 = -23.714 \pm 0.013$ fm is the NN ${}^1S_0$ scattering length, and $\gamma = \sqrt{M_N B}$. This expression for Y yields the $n + p \to d + \gamma$ capture cross section

$$\sigma = \frac{8\pi\alpha\gamma^5\kappa_1^2 a_0^2}{v M_N^5}\left(1 - \frac{1}{\gamma a_0}\right)^2 , \tag{6}$$

where α is the fine structure constant and v is the magnitude of the neutron velocity (in the proton rest frame). This agrees with the results of Bethe and Longmire [15,16] when terms in their expression involving the effective range are neglected. Eq. (6) is about 10% less than the experimental value for the cross section. In the power counting appropriate to the effective field theory approach the effective range enters at next order in the Q counting. However, other effects also occur at this order. For example, a two body operator involving the magnetic field. Including just the effective range does not represent a systematic improvement of the theoretical expression in eq. (6). This strong interaction amplitude was also examined in ref. [17] using effective field theory methods.

In terms of the amplitudes X, Y and W the parity violating asymmetry is,

$$A_\gamma = -\frac{2M_N}{\gamma^2}\, \frac{\mathrm{Re}[(Y+X)^*\, W]}{2|X|^2 + |Y|^2} \quad , \tag{7}$$

where γ^2/M_N is the photon energy. At leading order in the Q expansion W follows from the sum of diagrams in Fig. (2). We find that

$$W = g_A\, h_{\pi NN}^{(1)}\, \frac{\sqrt{\pi\gamma}}{3\pi f_\pi}\left[\frac{m_\pi}{(m_\pi+\gamma)^2} - \frac{m_\pi^2}{2\gamma^3}\ln\left(\frac{2\gamma}{m_\pi}+1\right) + \frac{m_\pi^2}{\gamma^2(m_\pi+\gamma)}\right] \quad , \tag{8}$$

where g_A is defined in eq. (3), $h_{\pi NN}^{(1)}$ is defined in eq. (2), and $m_\pi \simeq 140$ MeV is the pion mass. The first term in the square brackets comes from the one loop diagrams while the second and third terms come from the two loop diagrams with the neutron and proton rescattering through a contact term. Therefore, at leading order in the effective field theory Q expansion the numerical value of the asymmetry A_γ is,

$$A_\gamma = +0.17\, h_{\pi NN}^{(1)} \quad . \tag{9}$$

A naive dimensional analysis estimate of the weak coupling [13] yields $|h_{\pi NN}^{(1)}| \sim 5 \times 10^{-7}$, arising largely from the strange quark operators [18], consistent with the best guess of DDH (a recent calculation in the $SU(3)$ Skyrme model yields $h_{\pi NN}^{(1)} \sim +1.3 \times 10^{-7}$ [19]). Hence an asymmetry $|A_\gamma| \sim 0.8 \times 10^{-7}$ could reasonably be expected. This is consistent with the present experimental bound and would be easily accessible to the experiment proposed in Ref. [1].

The calculated asymmetry in eq. (9) is somewhat larger in magnitude than previous calculations that have found $A_\gamma \sim -0.11\, h_{\pi NN}^{(1)}$ [20–24]. It is also of the opposite sign to the

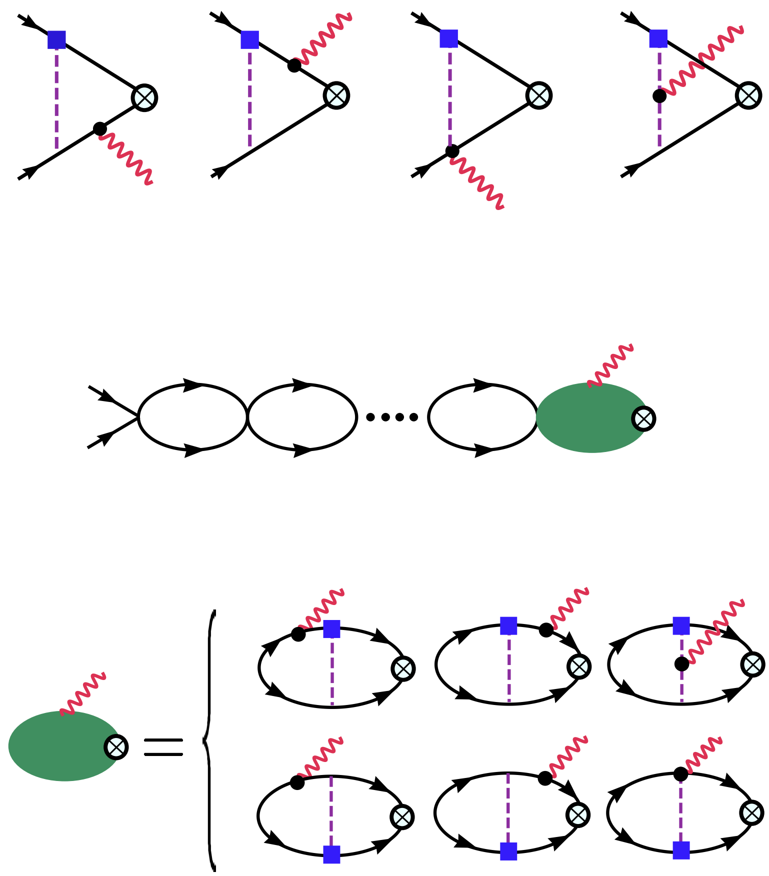

FIG. 2. *Graphs contributing to the parity violating amplitude for* $n+p\rightarrow d+\gamma$ *at leading order in the effective field theory expansion. The solid lines denote nucleons, the dashed lines denote pions and the wavy lines denote photons. The solid squares denote an insertion of* $\mathcal{L}_{pnc}$ *(eq. (2)) while the solid circles correspond to the minimal electromagnetic coupling. The crossed circle represents an insertion of the deuteron interpolating field which is taken to have* $^3S_1$ *quantum numbers.*

currently accepted theoretical prediction [23]. However, as stated in a footnote in [23], the sign of the asymmetry computed in [23] disagrees with the sign computed in [20,24].

The Feynman diagrams that contribute to W contain a contribution from exchange currents where the photon couples to the exchanged pion. This contribution by itself is ultraviolet divergent, yet the sum of diagrams is finite. This means that the value of the exchange current contribution alone is dependent on the ultraviolet regulator and subtraction scheme adopted. In potential models, the short distance behavior of the potential regulates the ultraviolet behavior. Many different models for the short distance behavior of the potential give the same low energy physics and the size of the exchange current contribution to W depends on how the short distance physics is modeled. For the parity conserving amplitude Y the exchange contribution does not occur until next-to-leading order in the Q expansion. A similar situation occurs there and again the value of the exchange current contribution alone is not a meaningful quantity since in the effective field theory approach that contribution is ultraviolet divergent and its value depends on the regulator and subtraction scheme used.

In the Q expansion the leading contribution to W is $\sim 1/\sqrt{Q}$. At next-to-leading order (i.e., $\sim \sqrt{Q}$) the parity violating S-wave to P-wave two body operators contribute. Since their coefficients are not known it is not possible at this time to improve the calculation of W by going beyond leading order in the Q expansion. However, the same coefficients appear in other parity violating observables so that a systematic analysis of higher order effects may be possible.

To conclude, we have computed the parity violating symmetry A_γ in the radiative capture of polarized cold neutrons $\vec{n} + p \rightarrow d + \gamma$ at leading order in effective field theory. The asymmetry A_γ will provide a relatively clean determination of $h^{(1)}_{\pi NN}$, unless this coupling is anomalously small, as suggested by the circular polarization experiments in ^{18}F [25]. If it is much smaller than naive estimates suggest then there will be additional contributions from parity violating two body operators that would need to be included.

We would like to thank Wick Haxton and Bruce McKellar for useful discussions. This work is supported in part by the U.S. Dept. of Energy under Grants No. DE-FG03-97ER4014 and DE-FG02-96ER40945.

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